



Rossmoyne Senior High School

Semester One Examination, 2024

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures

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Circle your teacher:

Ms Alvaro	Ms Lee
Mr Liu	Ms Robinson

In words

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Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	93	65
Total					100

Instructions to candidates

- The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
8	8	
9	6	
10	7	
11	6	
12	8	
13	8	
14	8	
15	10	
16	7	
17	8	
18	9	
19	8	
Section 2 Total	93	
Weighted ($\times 0.6915$)	65%	

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Section Two: Calculator-assumed

65% (93 Marks)

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

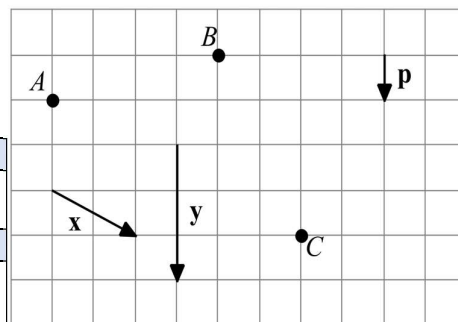
Question 8

(8 marks)

Two points, A and B , as well as two vectors, $\vec{x}$ and $\vec{y}$, are shown on the grid on the right

- (a) Write down a vector in terms of $\vec{x}$ and $\vec{y}$ that gives $\vec{AB}$. (2 marks)

Solution
$\vec{AB} = 2\vec{x} - \vec{y}$
Specific behaviours
✓ Correct scalar multiple of $\vec{x}$.
✓ $-\vec{y}$ and expression for $\vec{AB}$.



- (b) (i) It is given that $\vec{AC} = 3\vec{x}$. Show the position of point C on the above grid. (1 mark)

Solution
See Above Diagram
Specific behaviours
✓ Plots point C correctly.

- (ii) Hence, determine a vector in terms of $\vec{x}$ and $\vec{y}$ that gives $\vec{BC}$. (2 marks)

Solution
$\vec{BC} = \vec{x} + \vec{y}$
Specific behaviours
✓ ✓ Correct expression for $\vec{BC}$.

The vector $\vec{p}$ is the vector projection of $\vec{x}$ onto $\vec{y}$.

- (c) (i) Draw, and clearly label, vector $\vec{p}$ on the above grid. (1 mark)

Solution
See Above Diagram
Specific behaviours
✓ Plots point C correctly.

- (ii) State $\vec{p}$ in terms of $\vec{x}$ and/or $\vec{y}$. (1 mark)

Solution
$\vec{p} = \frac{1}{3}\vec{y}$
Specific behaviours
✓ Correct scalar multiple of $\vec{x}$ and/or $\vec{y}$ consistent with part (c)(i).

- (iii) Hence, determine the vector projection of $4\vec{x}$ onto $2\vec{y}$ in terms of $\vec{p}$. (1 mark)

Solution
$4\vec{p}$
Specific behaviours
✓ Correct scalar multiple of $\vec{p}$.

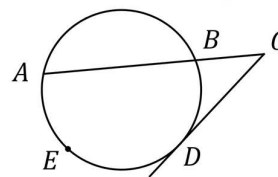
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Question 9

UNIT QUESTION: Deduct one mark for missing units in any part

(6 marks)

- (a) In the diagram (not drawn to scale), the tangent to the circle at D meets the extension of chord AB at C , and E lies on the arc AD , where $AEDB$ is a major segment of the circle. The radius of the circle is 3.3 cm, $AB = 4.5$ cm and $CD = 5.4$ cm.



- (i) Determine the length BC .

(2 marks)

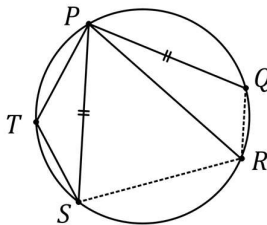
Solution
Using intersecting secant-tangent property, $CD^2 = CB \times CA$.
Hence $5.4^2 = CB(CB + 4.5) \Rightarrow CB = 3.6$ cm.
Specific behaviours
✓ uses secant-tangent property to form equation
✓ correct length INCLUDING UNIT

- (ii) Determine, to the nearest degree, the size of $\angle AEB$.

(2 marks)

Solution
Angle on circumference is half angle at centre:
$\angle AEB = \frac{1}{2} \angle AOB = \frac{1}{2} \left(2 \sin^{-1} \left(\frac{4.5 \div 2}{3.3} \right) \right) = 43.0^\circ$
Specific behaviours
✓ expression for $\angle AOB$
✓ correct angle

- (b) The points P, Q, R, S and T lie in that order on the circumference of a circle, so that $PQ = PS$ and $\angle PRQ = 37^\circ$. Determine, with justification, the size of $\angle PTS$. (2 marks)

Solution	
	Chords of equal length subtend equal angles and so $\angle PRS = \angle PRQ = 37^\circ$. $PRST$ is a cyclic quadrilateral and so $\angle PTS = 180^\circ - \angle PRS = 180^\circ - 37^\circ = 143^\circ$.
Specific behaviours	
✓ justifies that $\angle PRS = \angle PRQ$	
✓ correct $\angle PTS$, with justification	

Question 10

(7 marks)

- (a) Anders is the recruitment manager for a building company and has just interviewed 85 candidates, of whom 44 were qualified carpenters, 52 were qualified electricians and 37 were qualified plumbers. Furthermore, 11 of the candidates were qualified in all three trades and all of them were qualified in at least one trade. Determine the number of candidates Anders hired, given that he hired all candidates qualified in at least two trades. (3 marks)

Solution
$n(C \cup E \cup P) = n(C) + n(E) + n(P) - n(C \cap E) - n(C \cap P) - n(E \cap P) + n(C \cap E \cap P)$ Let $x = n(C \cap E) + n(C \cap P) + n(E \cap P)$ so that $85 = 44 + 52 + 37 - x + 11 \Rightarrow x = 59$. But x includes the 11 three times, so Anders hired $59 - 2(11) = 37$ candidates.
Specific behaviours
✓ indicates correct use of inclusion-exclusion principle ✓ indicates correct number for x ✓ correct number hired

- (b) Determine how many of the integers between 1 and 2024 inclusive are

- (i) divisible by 42.

(1 mark)

Solution
$\lfloor 2024 \div 42 \rfloor = 48 \text{ integers.}$
Specific behaviours
✓ correct number

- (ii) divisible by 14 or 21 but not both.

(3 marks)

Solution
Integers divisible by each: $\lfloor 2024 \div 14 \rfloor = 144$, $\lfloor 2024 \div 21 \rfloor = 96$. Noting that LCM of numbers is 42 and using inclusion-exclusion principle, number divisible by one or the other or both is $144 + 96 - 48 = 192$. Hence number divisible by one or the other but not both is $192 - 48 = 144$.
Specific behaviours
✓ indicates correct count of numbers divisible by 14, 21 ✓ uses inclusion-exclusion to obtain count of 14 or 21 ✓ correct number

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Question 11

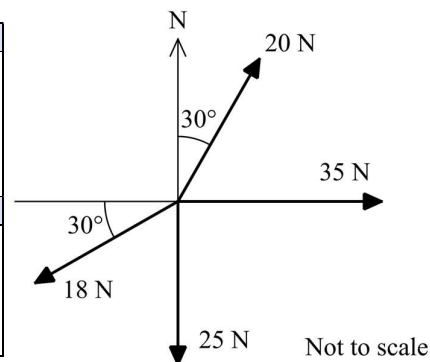
(6 marks)

The diagram on the right shows four forces acting on a body.

All the forces are in the same plane.

- (a) Find the magnitude and bearing of the resultant, *correct to two decimal places*. (4 marks)

Solution 1	
20N vector: $10\mathbf{i} + 10\sqrt{3}\mathbf{j}$,	35N vector: $35\mathbf{i}$
25N vector: $-25\mathbf{j}$,	18N vector: $-9\sqrt{3}\mathbf{i} - 9\mathbf{j}$
$\mathbf{r} = (-9\sqrt{3} + 45)\mathbf{i} + (10\sqrt{3} - 34)\mathbf{j}$	
Magnitude = 33.81 N, Bearing = 119.56°	
Specific behaviours	
✓ Converts vectors to component form. ✓ Determines resultant vector. ✓ Determines magnitude. ✓ Determines bearing.	



Solution 2	
Using technology:	
$[20, \angle 60] + [35, \angle 0]$	
$+ [25, \angle (-90)] + [18, \angle (-150)]$	
$= (-9\sqrt{3} + 45)\mathbf{i} - (10\sqrt{3} - 34)\mathbf{j}$	
Magnitude = 33.81 N	
Bearing = 119.56°	
Specific behaviours	
✓ Converts vectors to component form. ✓ Determines resultant vector. ✓ Determines magnitude. ✓ Determines bearing.	

- (b) Find the magnitude and bearing of the single force that will keep this system in equilibrium, *correct to two decimal places*.

(2 marks)

Solution	
Vector is in opposite direction:	
Magnitude = 33.81 N	
Bearing = 299.56°	
Specific behaviours	
✓ Recognises vector is in opposite direction, and has same magnitude. ✓ Determines bearing.	

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Question 12

(8 marks)

The heights of players in a competitive basketball squad are categorised as tall, medium or short, and of the 16 players in the squad, 6 are tall, 7 medium and the rest short.

- (a) When players are randomly selected from the squad, determine, the least number of players that must be picked to be certain that at least 5 players with the same height category are chosen. (2 marks)

Solution
Pick 3 short, 4 medium and 4 tall. The next pick will ensure at least 5 mediums or tall. Hence 12 players must be selected.
Specific behaviours
✓ justification ✓ correct number

- (b) Determine the number of distinct ways that 5 players can be chosen from the squad

- (i) with no restrictions. (1 mark)

Solution
$\binom{16}{5} = 4368$ ways.
Specific behaviours
✓ correct number of ways

- (ii) so that exactly one player is short. (1 mark)

Solution
$\binom{3}{1} \binom{13}{4} = 3 \times 715 = 2145$ ways.
Specific behaviours
✓ correct number of ways

- (iii) so that at least half of the players are tall. (2 marks)

Solution
$\binom{6}{5} + \binom{6}{4} \binom{10}{1} + \binom{6}{3} \binom{10}{2} = 6 + 150 + 900 = 1056$ ways.
Specific behaviours
✓ correct number for one possible case ✓ correct number of ways

- (c) The name of the coach of the squad is ANNEMARIEKA. Determine the number of distinct arrangements of all 11 letters in her name with the vowels and consonants alternating. (2 marks)

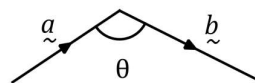
Solution											
Require vowel-consonant-vowel, etc, and note $3 \times A$, $2 \times N$, and $2 \times E$:											
Type	<i>v</i>	<i>c</i>	<i>v</i>	<i>c</i>	<i>v</i>	<i>c</i>	<i>v</i>	<i>c</i>	<i>v</i>	<i>c</i>	<i>v</i>
Choices	6	5	5	4	4	3	3	2	2	1	1
Hence there are $\frac{6!5!}{3!2!2!} = 3600$ arrangements.											
Specific behaviours											
✓ indicates number of alternating arrangements											
✓ adjust for duplicates to obtain correct number											

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Question 13

(8 marks)

- (a) Determine the exact scalar product of the two vectors shown (not to scale), given that $\theta = 150^\circ$, $|\underline{a}| = 6$ and $|\underline{b}| = 15$.



(2 marks)

Solution
Angle between directions is $180^\circ - 150^\circ = 30^\circ$.
Scalar product is $15 \times 6 \times \cos(30^\circ) = 45\sqrt{3}$.
Specific behaviours
✓ correct angle between directions
✓ obtains scalar product

- (b) Let $\underline{r} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$, $\underline{s} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$ and $\underline{t} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$, where $\underline{t} = \lambda \underline{r} + \mu \underline{s}$. Determine

- (i) $4\underline{r} - 7\underline{s} + 3\underline{t}$.

(1 mark)

Solution
$4\underline{r} - 7\underline{s} + 3\underline{t} = \begin{pmatrix} 66 \\ -48 \end{pmatrix}$
Specific behaviours
✓ correct vector

- (ii) the angle between the directions of $\underline{r}$ and $\underline{s}$.

(2 marks)

Solution
Using CAS, the angle is 149° .
Specific behaviours
✓ indicates suitable method
✓ correct angle

- (iii) the value of the constant λ and the value of the constant μ .

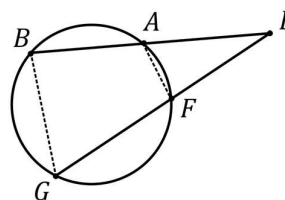
(3 marks)

Solution
$\begin{pmatrix} 5 \\ -3 \end{pmatrix} = \lambda \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 5 \end{pmatrix}$
$5 = 4\lambda - 5\mu, \quad -3 = -\lambda + 5\mu$
Solving simultaneously, $\lambda = 2/3$ and $\mu = -7/15$.
Specific behaviours
✓ forms equation using i-coefficients
✓ forms equation using j-coefficients
✓ solves equations to obtain correct values

Question 14

(8 marks)

The diagram shows two secants drawn through the same circle from point E that is external to the circle.



One secant from point E meets the circle first at point A and then at point B and the other secant meets the circle first at point F and then at point G .

- (a) Prove that triangle AEF is similar to triangle EGB .

(4 marks)

Solution
Angle E is common to both triangles. $\angle EAF = 180^\circ - \angle BAF$ (angle sum on straight line) $\angle EGB = 180^\circ - \angle BAF$ (opposite angles in cyclic quadrilateral) $\therefore \angle EAF = \angle EGB$. Hence $\triangle AEF \sim \triangle EGB$ (corresponding angles equal).
Specific behaviours
✓ states common angle in both triangles ✓ uses angle sum on straight line with $\angle BAF$ (or $\angle AFG$) ✓ uses cyclic quadrilateral with $\angle BAF$ (or $\angle AFG$) ✓ deduces, with reasons, that triangles are similar

- (b) Show that $EA \cdot EB = EF \cdot EG$.

(1 mark)

Solution
Using ratio of corresponding sides: $\frac{EA}{EG} = \frac{EF}{EB} \rightarrow EA \cdot EB = EF \cdot EG$
Specific behaviours
✓ shows use of corresponding sides

- (c) Determine all possible lengths of EB when $AB = 34$ cm, EF is 4 cm longer than AE , and $FG = EF$.

(3 marks)

Solution
Let $EA = x$ $EA \cdot EB = EF \cdot EG$ $x(x + 34) = (x + 4) \times 2(x + 4)$ $x = 2, x = 16$ $EB = 34 + 2 = 36, EB = 34 + 16 = 50$ Hence EB is 36 or 50 cm.
Specific behaviours
✓ correctly forms quadratic equation ✓ obtains both solutions to quadratic equation ✓ correctly states both lengths

Question 15

(10 marks)

$\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 2$ and $|\vec{b}| = 3$. The angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$.

Two other vectors are defined as $\vec{u} = \lambda\vec{a} + \vec{b}$ and $\vec{v} = \vec{a} + \lambda\vec{b}$, such that $\lambda > 0$.

(a) Show that $|\vec{u}| = \sqrt{4\lambda^2 + 6\lambda + 9}$.

(3 marks)

Solution	
	$ \vec{u} ^2 = 3^2 + 4\lambda^2 - 2 \times 3 \times 2\lambda \cos \frac{2\pi}{3}$ $ \vec{u} ^2 = 4\lambda^2 + 6\lambda + 9$ $ \vec{u} = \sqrt{4\lambda^2 + 6\lambda + 9}$
Specific behaviours	
✓ Draws a diagram or indicates that when vectors are placed head to tail, the angle is $\frac{2\pi}{3}$. ✓ Uses cosine rule to determine magnitude of $\vec{u}$. ✓ Substitutes in magnitudes of $\lambda\vec{a}$ and $\vec{b}$ and arrives at required result.	

(b) Determine the expression for $|\vec{v}|$, similar to the form in Part (a).

(3 marks)

Solution	
	$ \vec{v} ^2 = 9\lambda^2 + 4 - 2 \times 3 \times 2\lambda \cos \frac{2\pi}{3}$ $ \vec{v} ^2 = 9\lambda^2 + 6\lambda + 4$ $ \vec{v} = \sqrt{9\lambda^2 + 6\lambda + 4}$
Specific behaviours	
✓ Uses cosine rule to determine magnitude of $\vec{v}$. ✓ Substitutes in magnitudes of $\vec{a}$ and $\lambda\vec{b}$ and angle of $\frac{2\pi}{3}$. ✓ Determines an expression for $\vec{v}$.	

(c) Determine the value(s) of λ , correct to 2 decimal places, such that the angle between $\vec{u}$ and $\vec{v}$ is $\frac{\pi}{6}$.

(4 marks)

Solution	
$\vec{u} \cdot \vec{v} = \vec{u} \vec{v} \cos \frac{\pi}{6}, \quad \text{LHS} = (\lambda\vec{a} + \vec{b}) \cdot (\vec{a} + \lambda\vec{b}) = \lambda \vec{a} ^2 + \lambda^2\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} + \lambda \vec{b} ^2$ $= 4\lambda + 3\lambda^2 + 3 + 9\lambda = 3\lambda^2 + 13\lambda + 3$ $\text{RHS} = \frac{\sqrt{3}}{2} \sqrt{4\lambda^2 + 6\lambda + 9} \sqrt{9\lambda^2 + 6\lambda + 4}$ $\text{Solving LHS} = \text{RHS on CAS: } \lambda = 0.35, 2.86$	
Specific behaviours	
✓ Uses dot product to write down an expression relating $\vec{u}$, $\vec{v}$ and angle between the vectors. ✓ Expands LHS out in terms of $\vec{a}$ and $\vec{b}$. ✓ Determines $\vec{a} \cdot \vec{b}$, and simplifies LHS. ✓ Solves for λ.	

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Question 16

(7 marks)

- (a) A plain pink shirt has five buttons, each of which must be one of seven different colours. Determine the number of different button arrangements on the shirt if repetition of colour

- (i) is allowed.

(1 mark)

Solution
$n = 7^5 = 16\,807$
Specific behaviours
✓ correct number

- (ii) is not allowed.

(1 mark)

Solution
$n = {}^7P_5 = 2520$
Specific behaviours
✓ correct number

- (b) When n objects arranged in a row are rearranged so that none of them occupy their original positions, the objects are said to be deranged. For example, BCA is a derangement of ABC but BAC is not because the C occupies its original position. The number of derangements D_n of n distinct objects is shown in the table below.

n	4	5	6	7	8	9
D_n	9	44	265	1854	14833	133496

- (i) List all the permutations of the letters A, B and C , circle the derangements of ABC and hence state the value of D_3 . (2 marks)

Solution
$ABC, ACB, BAC, \textbf{BCA}, \textbf{CAB}, CBA \rightarrow D_3 = 2$
Specific behaviours
✓ correctly lists all permutations
✓ correct value of D_3

- (ii) Given 10 people and their passport photos, determine the number of ways in which the people can be matched with the photos so that exactly 4 of them have the correct photo. (3 marks)

Solution
Correctly match 4 people in $\binom{10}{4} = 210$ ways.
Incorrectly match remaining 6 people in $D_6 = 265$ ways.
Matching can be done in $210 \times 265 = 55\,650$ ways.
Specific behaviours
✓ indicates use of combination for correct matches
✓ indicates correct method for derangements
✓ correct number of ways

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Question 17

(8 marks)

PQ and RS are two intersecting chords in the same circle and RS is parallel to the tangent to the circle at P .

- (a) Sketch a diagram to show this information. (1 mark)

Solution
Specific behaviours
✓ correctly shows circle, 3 solid lines, 4 given points, parallel indicators

- (b) Prove that PQ bisects $\angle RQS$. (4 marks)

Solution
Let A be a point on the tangent as shown. $\angle PSR = \angle SPA$ (alternate angles on PS) $\angle PQR = \angle PSR$ (angles on arc PR) and so $\angle PQR = \angle SPA$ $\angle PQS = \angle SPA$ (alternate segments on PS) Hence $\angle PQS = \angle PQR = \angle SPA$ and so PQ bisects $\angle RQS$. <i>(R and S are interchangeable - see their diagram)</i>
Specific behaviours
✓ shows $\angle PSR = \angle SPA$ with reason ✓ shows $\angle PQR = \angle PSR$ with reason ✓ shows $\angle PQS = \angle SPA$ with reason ✓ writes and completes proof in logical manner

- (c) In the case when RS is a diameter, the acute angle between PQ and RS is 67° and the length of minor arc QR is less than the length of minor arc QS , determine the size of $\angle QOR$, where O is the centre of the circle. Justify your answer. (3 marks)

Solution
Let PQ and RS intersect at B so that $\angle QBR = 67^\circ$. $\angle PQS = 90^\circ \div 2 = 45^\circ$ (from part (a) and angle in semicircle) $\angle QSR + 45^\circ = 67^\circ \Rightarrow \angle QSR = 22^\circ$ (interior-exterior angle in Δ) $\angle QOR = 2 \times 22^\circ = 44^\circ$ (angles at centre, circumference) <i>(Alternative approaches exist, such as ΔQOR is isosceles, etc.)</i>
Specific behaviours
✓ $\angle PQS$ with reason ✓ $\angle QSR$ with reason ✓ correct $\angle QOR$

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Question 18

(9 marks)

The navigator of a small pilot boat with a cruising speed of 12.5 km/h must calculate the required constant velocity vector $\begin{pmatrix} x \\ y \end{pmatrix}$ km/h the boat should steer from port P to ship S . Relative to a nearby lighthouse L , $\overrightarrow{LP} = \begin{pmatrix} 0.91 \\ -1.22 \end{pmatrix}$ km and $\overrightarrow{LS} = \begin{pmatrix} 2.19 \\ 4.08 \end{pmatrix}$ km, and a steady current of $\begin{pmatrix} -1.20 \\ 1.55 \end{pmatrix}$ km/h flows in the local waters.

- (a) Explain why $x^2 + y^2 = 156.25$. (1 mark)

Solution
The speed of 12.5 is the magnitude of the velocity vector and so $x^2 + y^2 = 12.5^2 = 156.25.$
Specific behaviours
✓ correct explanation

- (b) Explain why $\begin{pmatrix} x - 1.2 \\ y + 1.55 \end{pmatrix} = \lambda \begin{pmatrix} 1.28 \\ 5.3 \end{pmatrix}$, where λ is a constant, and hence deduce another relationship between x and y . (4 marks)

Solution
Boat and water move with velocity $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -1.20 \\ 1.55 \end{pmatrix} = \begin{pmatrix} x - 1.2 \\ y + 1.55 \end{pmatrix}$ and as this vector must be in the same direction as $\overrightarrow{PS} = \begin{pmatrix} 2.19 \\ 4.08 \end{pmatrix} - \begin{pmatrix} 0.91 \\ -1.22 \end{pmatrix} = \begin{pmatrix} 1.28 \\ 5.3 \end{pmatrix}$ then one vector must be a multiple of the other.
$x - 1.2 = 1.28\lambda \rightarrow \lambda = \frac{x - 1.2}{1.28}, \quad y + 1.55 = 5.3\lambda \rightarrow \lambda = \frac{y + 1.55}{5.3}$ $\frac{x - 1.2}{1.28} = \frac{y + 1.55}{5.3}$
Specific behaviours
✓ derives velocity vector and displacement vector $\overrightarrow{PS}$ ✓ explains one vector must be a multiple of the other ✓ equates components ✓ eliminates λ to obtain correct equation

- (c) Using CAS, or otherwise, determine the required velocity vector $\begin{pmatrix} x \\ y \end{pmatrix}$ for the boat. (2 marks)

Solution
Solving the equations $x^2 + y^2 = 156.25$ and $\frac{x-1.2}{1.28} = \frac{y+1.55}{5.3}$ simultaneously with CAS and ignoring solution set in opposite direction we get $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4.4 \\ 11.7 \end{pmatrix}$.
Specific behaviours
✓ value of x ✓ value of y

- (d) At what time, to the nearest minute, would the pilot boat reach ship S if it left port P at 7:48 am? (2 marks)

Solution
Time $t = 1.28 \div (4.4 - 1.2) = 0.4$ h = 24 m and so boat will reach ship at 8:12 am.
Specific behaviours
✓ indicates use of a component to calculate journey time ✓ determines journey time and time of arrival

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ROUNDING QUESTION: Deduct once if not rounding to required decimal places.

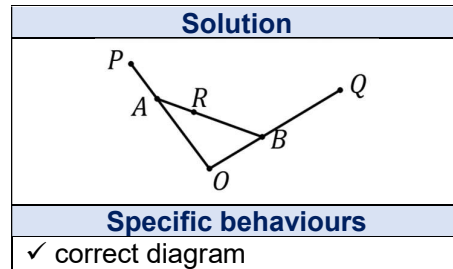
Question 19

(8 marks)

Relative to the origin O , points P and Q have position vectors $\vec{p}$ and $\vec{q}$ respectively. Point A lies on OP so that $\vec{OA} = k\vec{p}$, point B lies on OQ so that $\vec{OB} = (1-k)\vec{q}$ and point R lies on AB so that $\vec{AR} = (1-k)\vec{AB}$, for some scalar constant k where $0 < k < 1$.

- (a) Sketch a diagram to show
- O
- and the other five points.

(1 mark)



- (b) Show that
- $\vec{OR} = k^2\vec{p} + (1-k)^2\vec{q}$
- .

(3 marks)

Solution
$\begin{aligned}\vec{OR} &= \vec{OA} + \vec{AR} \\ &= \vec{OA} + (1-k)\vec{AB} \\ &= \vec{OA} + (1-k)(\vec{OB} - \vec{OA}) \\ &= \vec{OA} - \vec{OA} + k\vec{OA} + (1-k)\vec{OB} \\ &= k\vec{OA} + (1-k)\vec{OB} \\ &= k^2\vec{p} + (1-k)^2\vec{q}\end{aligned}$
Specific behaviours
✓ obtains expression for $\vec{AR}$ in terms of $\vec{OA}$ and $\vec{OB}$ ✓ expands and simplifies expression for $\vec{OR}$ in terms of $\vec{OA}$ and $\vec{OB}$ ✓ substitutes for $\vec{OA}$ and $\vec{OB}$ to obtain result

Let d be the distance of R from the origin when $\vec{p} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$.

- (c) Show that
- $d = \sqrt{9(1-k)^4 + 16k^4}$
- .

(2 marks)

Solution
$\begin{aligned}\vec{OR} &= k^2 \begin{pmatrix} 0 \\ 4 \end{pmatrix} + (1-k)^2 \begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} -3(1-k)^2 \\ 4k^2 \end{pmatrix} \\ d &= \vec{OR} = \sqrt{(-3(1-k)^2)^2 + (4k^2)^2} \\ &= \sqrt{9(1-k)^4 + 16k^4}\end{aligned}$
Specific behaviours
✓ obtains expression for $\vec{OR}$ in terms of k ✓ clearly derives expression for distance

- (d) By graphing
- d
- against
- k
- , or otherwise, determine the minimum value of
- d
- and the value of
- k
- required to achieve this minimum, giving your answers correct to the nearest 0.001.

(2 marks)

Solution
The least distance is 1.216 when $k = 0.452$.
Specific behaviours
✓ distance ✓ correct value of k with PROPER ROUNDING

Supplementary page

Question number: _____

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